5.3 Multiple Angle Formulae :

Consider

$$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$$

If $A = B$

$$\sin 2A = \sin A.\cos A + \cos A.\sin A$$

$$\boxed{\sin 2A = 2\sin A.\cos A} \quad ...($$

Consider

$$\cos(A+B) = \cos A.\cos B - \sin A.\sin B$$

If $A = B$

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A} \quad ...(2)$$

From above equation

$$\cos 2A = \cos^2 A - (1 - \cos^2 A) \text{ ..... } [\sin^2 A = 1 - \cos^2 A]$$

$$\cos 2A = \cos^2 A - 1 + \cos^2 A$$

$$\boxed{\cos 2A = 2\cos^2 A - 1} \quad ...(3)$$

Similarly

$$\cos 2A = (1 - \sin^2 A) - \sin^2 A \qquad [\cos^2 A = 1 - \sin^2 A]$$

$$\cos 2A = (1 - \sin^2 A) - \sin^2 A$$

$$\boxed{\cos 2A = 1 - 2\sin^2 A} \quad ...(4)$$

Consider

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

If $A = B$

$$\tan 2A = \frac{\tan A + \tan A}{1 - \tan A.\tan A}$$

$$\boxed{\tan 2A = \frac{2\tan A}{1 - \tan^2 A}} \quad ...(5)$$

Now

$$\sin 3A = \sin(2A + A)$$

$$\sin 3A = \sin 2A.\cos A + \cos 2A.\sin A \quad \begin{cases} \sin 2A = 2\sin A.\cos A \\ \cos 2A = 1 - 2\sin^2 A \end{cases}$$

$$\sin 3A = 2\sin A.\cos A.\cos A + (1 - 2\sin^2 A).\sin A$$

$$\sin 3A = 2\sin A.\cos^2 A + \sin A - 2\sin^3 A$$

$$sin3A = 2sinA(1-sin^2A)+sinA-2sin^3A \quad cos^2A = 1-sin^2A$$

$$sin3A = 2sinA - 2sin^3A + sinA - 2sin^3A$$

$$\boxed{sin3A = 3sinA - 4sin^3A} \quad ...(6)$$

Again consider

$$cos3A = cos(2A + A)$$

$$cos3A = cos2A.cosA - sin2A.sinA$$

$$cos3A = (2cos^2A - 1)cosA - 2sinA.cosA.sinA$$

$$cos3A = 2cos^3A - cosA - 2cosA.sin^2A$$

$$cos3A = 2cos^3A - cosA - 2cosA(1 - cos^2A)$$

$$cos3A = 2cos^3A - cosA - 2cosA + 2cos^3A$$

$$\boxed{cos3A = 4cos^3A - 3cosA} \quad ...(7)$$

Similarly

$$tan3A = tan(2A + A)$$

$$tan3A = \frac{tan2A + tanA}{1 - tan2A.tanA}$$

$$tan3A = \frac{\frac{2tanA}{(1-tan^2A)} + tanA}{1 - \frac{2tanA}{(1-tan^2A)}.tanA}$$

$$tan3A = \frac{2tanA + tanA(1-tan^2A)}{(1-tan^2A) - 2tan^2A}$$

$$tan3A = \frac{2tanA + tanA - tan^3A}{1 - tan^2A - 2tan^2A}$$

$$\boxed{tan3A = \frac{3tanA - tan^3A}{1 - 3tan^2A}} \quad ...(8)$$

5.4 Worked Examples on Multiple Angles :

1. Show that, i) $sin^2 A = \frac{1-cos 2A}{2}$ ii) $cos^2 A = \frac{1-cos 2A}{2}$

Solution :

i)
$$cos2A = cos^2 A - sin^2 A$$
$$cos2A = (1 - sin^2 A) - sin^2 A \quad (\because cos^2 A = 1-sin^2 A)$$
$$cos2A = 1 - 2 sin^2 A$$
$$2 sin^2 A = 1 - cos2A$$
$$sin^2 A = \frac{1-cos 2A}{2}$$

ii) We have
$$cos2A = cos^2 A - sin^2 A$$
$$cos2A = cos^2 A - (1 - cos^2 A) \quad (\because sin^2 A = 1-cos^2 A)$$
$$cos2A = 2 cos^2 A - 1$$
$$2cos^2 A = 1 + cos2 A$$
$$cos^2 A = \frac{1+cos 2A}{2}$$

2. Prove that $cos^4\theta - sin^4\theta = cos2\theta$ (Nov./Dec. 2018)

Solution : LHS $= cos^4\theta - sin^4\theta$

$= (cos^2\theta + sin^2\theta)(cos^2\theta - sin^2\theta)$ [Functorise]

$= (cos^2\theta - sin^2\theta)$ $[cos^2\theta + sin^2\theta = 1]$

$= cos2\theta$ = RHS

3. If $cosA = \frac{1}{2}$ find $cos2A$ (May 2014)

Solution : $cos2A = 2cos^2 A - 1$

$$= 2 \times \left(\frac{1}{2}\right)^2 - 1$$

$$= \frac{1}{2} - 1 = -\frac{1}{2}$$

4. If $\sin A = \frac{1}{2}$ find the numerical value of Sin2A (Nov./Dec. 201

Solution : $\sin A = \frac{1}{2}$ $\quad \cos A = \sqrt{1 - \sin^2 A}$

$$\cos A = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \sin 2A = 2 \sin A . \cos A$$

$$\sin 2A = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

5. If $\sin\theta = \frac{3}{5}$ find the value of $\sin 2\theta$ (Nov./Dec. 20

Solution : $\sin\theta = \frac{3}{5}, \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{9}{25}} =$

$$\sin 2\theta = 2\sin\theta . \cos\theta$$

$$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

7. Prove that $\frac{1+\cos 2x}{\sin 2x} = \cot x$ (April/May 2017)

Solution : LHS $= \frac{1+\cos 2x}{\sin 2x}$ $(\cos 2x = 2\cos^2 x - 1)$

$= \frac{2\cos^2 x}{2\sin x \cos x}$

$= \cot x =$ RHS

8. Show that $\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \tan\theta$ (May 2006)

Solution : LHS $= \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \sqrt{\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}}$

$= \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = \frac{\sin\theta}{\cos\theta}$

$= \tan\theta =$ RHS

9. Prove that $\tan A + \cot A = 2\operatorname{cosec}(2A)$

Solution : $LHS = \tan A + \cot A$

$LHS = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$LHS = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$

$LHS = \frac{2 \times 1}{2\sin A \cos A}$

$LHS = \frac{2}{\sin 2A} = 2\operatorname{cosec} 2A =$ RHS